PROCEEDINGS OF THE SYMPOSIUM* ON "APPLICATIONS OF ROBUST NON-PARAMETRIC INFERENCE TECHNIQUES IN AGRICULTURE"

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After the Chairman's opening remarks, the speakers presented their papers.

Professor J.V. Deshpande presented Wilks' classical method for setting up a non-parametric confidence interval for quantiles and followed it up with modern rank-based confidence intervals for such parameters as location of a symmetry, difference in location parameters and scale parameters. This second set of results (begun in the sixties) is due mainly to Hodges and Lehmann. Professor Deshpande pointed out that in many non-parametric problems, comparing location parameters of X and Y may not be as meaningful as determining confidence intervals for the parameter $P(X<Y)$.

Professor S.K. Chatterjee reviewed non-parametric rank tests for testing significance of treatment effects in the context of randomized block and incomplete block experiments. He considered two situations e.g. (*i*) when the errors within blocks have an exchangeable distribution, and (*ii*) all the errors in the experimental set-up have an exchangeable distribution. If one suspects not merely lack of normality but heterogeneity, then assumption (*i*) would be appropriate. If homoscedesticity is expected but not normality then (*ii*)

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would be more appropriate. He referred to tables in Owen and Hollander and Wolfe. He also pointed out that for the majority of applications in agriculture in India, the number of blocks is usually 3 or 4 and the number of treatments is about 10. For these values, neither the existing tables nor the asymptotic approximations are available. He also mentioned about some new work under progress on making better use of data than rank tests do in the presence of additional information.

Shri S.C. Rai reviewed some of the literature on tests based on nominal and ordinal scales and also discussed models for rank analysis in paired and triad comparisons. He also presented some of his own tests and their applications.

Dr. T.J. Khatri presented a table showing, to what extent the classical F-test and Friedman's rank test lead to the same decisions; this has been studied for different levels of significance and using data arising out of 125 actual agricultural experiments. They show that the agreement is on the whole satisfactory.

Dr. R. Singh mentioned about some applications of non-parametric tests in situations where the frame is not available separately for different classes to be compared and so the sample size falling in each class is random and not very large.

Dr. V.K. Bhatia demonstrated the use of non-parametric survival analysis applied to culling times in studying the yield-survival relationship in dairy cattle. This is the first time that the yield-survival relationship has been studied this way in the Indian context. The idea of culling as a kind of mortality is new; this has made possible the application of actual techniques, Cox's regression models and the Mantel-Haenszel estsmate. Since the 'Life' of a cow is being measured in terms of the number of successfully completed lactations before culling, some modifications of Cox's technique had to be used. Detailed numerical studies have been made of the survival time distribution of different breeds.

Discussion :

During the discussions on these papers, the following points emerged :

(*i*) Though good non-parametric alternatives to classical tests exist, further investigation is needed before existing methods are

completely replaced by non-parametric procedures. Both from the point of view of Bayesian analysis and the likelihood principle, the classical tests come out rather well. But the rank tests have not been studied from this point of view. In this context, reference may be made to Dr. Basu (JASA, 1980) and Bickel (Scandanavian Journal of Statistics). On the other hand, the findings of Khatri, referred to above were re-assuring. However, it may be pointed out the Khatri had used X^2-approximation whenever tables for Friedman's test were not available.

(*ii*) One of the biggest handicaps in applying the rank tests is lack of tables or computer programmes as pointed out by Professor S.K. Chatterjee.

(*iii*) It was also pointed out that most non-parametric procedures for setting up confidence intervals is based on the assumption that treatments change only the location parameter ; sometimes also a symmetric distribution of errors is needed. It would be desirable to check these through uniformity trials and other controlled experiments.

(*iv*) It was felt that Dr. Bhatia's analysis of culling data should bring in the notion of censoring. Indeed Cox's model is quite suitable for handling such problems.

(*v*) Rank tests are less sensitive to outliers than the classical tests. However, techniques based on a narrower definition of robustness due to Huber may be more appropriate in this situation.

Recommendations :

There should be further studies, through symposia and discussions, of the following nature :

1. Non-parametric methods, particularly to the extent to which they should replace the existing methods.
2. Bayesian and Likelihood approaches to ANOVA, to the extent one can judge performance of rank tests and other non-parametric tests.
3. Robustness, in the sense of Huber, particularly in the context of regression.
4. Survival analysis with special emphasis on Cox models and partial likelihood.

The detailed summaries of the papers are as follows.

1. Distribution-free Confidence Intervals and Point Estimation

BY

J.V. DESHPANDE

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This is a review papar to introduce techniques of point and interval estimation without the knowledge of the specific probability distribution from which the observations have arisen. The classical methodology is based on the assumption of normal or some other parametric family of distributions. However, there is a great need for methodology to deal with situations where normality is not a realistic assumption. In the literature confidence intervals for unknown quantiles under the very broad assumption *any* continuous distribution F are well known for many years. Perhaps less known are the confidence intervals for differences in locations, ratio of scales, linear contrasts in treatment effects in data classified in two or more ways, which are valid under the same broad assumption. We review the development in this and related areas and bring out the usefulness of these methods in agricultural experimentation by relevant examples.

2. Use of Nonparametric Procedures in the Analysis of Designs

BY

S.K. CHATTERJEE

Calcutta University, Calcutta

The analysis of designed experiments generally involves three principal steps : (*a*) application of an initial test to judge whether the treatments differ at all, (b) the identificattion of the significant treatment contrasts if the evidence of the initial test is positive, and (*c*) the setting up of point and interval estimates of the significant contrasts Standard parametric analysis is valid and optimal only under stringent assumptions about the joint distribution of the yields. These assumptions can conveniently be represented as a gradually tightening sequence of conditions as follow : (*i*) interchangeability of the yields within each block under the hypothesis of no treatment effects, or equivalently, interchangeability of the errors within each block, (*ii*) additivity of block effects, (*iii*) independence of the error-vectors corresponding to different blocks, (*iv*) interchangeability of the error vectors (*v*) independence of the errors within each block (*vi*) absoulte continuity of the distribution of errors, (*vii*) existence of lower order moments of errors, (*viii*) normality of errors. In

robustness studies we remove some of the conditions from the tail of the sequence and examine the properties of existing or newly proposed procedures under the ensuing broader set-ups. Two types of nonparametric procedures for analysing block designs are available in the literature. The first type-the within-block ranking procedures-hinges essentially on condition (*i*) The second type-the across-the-blockranking procedures-requires at least the conditions (*i*)-(*iii*). In this paper the use of different procedures in eteps (*a*), (*b*), and (*c*) of the analysis, roles of the different conditions in ensuring various properties of the procedures, and the limitations of the procedures are critically reviewed.

3. Non-Parametric Inference in Designed Experiments

BY

S.C. RAI,

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Statistics provides tools which formalise and standardise the procedures for drawing conclusions. The logic of the procedures of statistical inference dictates some of the conditions under which the evidence must be collected and statistical tests determine how large the observed differences must be before we can have confidence that they represent real differences in the larger group from which the data are taken. There are large number of techniques of inference which do not make numerous or stringent assumptions about parameters. These techniques are known as "distribution-free" or "non-parametric" techniques and they result in conclusions which require fewer qualifications.

There are broadly the following four scales of measurements in which the data can be recorded :

1. *Nominal Scale* : Numbers or other Symbols are used to identify the group to which various objects belong.

2. *Ordinal Scale* : Here the relationship of thc type 'higher than' or 'more prefarred' may be reflected.

3. *Interval Scale* ; It has all the properties of the ordinal scale and in addition, the distances between any two numbers on the scale are of known size.

4. *Ratio Scale* : It has all the properties of interval scale and in addition has a true zero point as its origin.

For two related samples, Mc.Nemar test, Sign test, Wilcoxon Matched-Pairs Signed-Ranks test and Randomisation test for matched pairs may be used. Keeping in view the applicability and efficiency of these tests, the following recommendations may be made.

Mc Nemar test for the significance of changes may be used when one or both of the conditions under study has been measured only in the sense of nominal scale. If the ordinal measurement within pairs is possible, then sign test may be used. When the measurement is in ordinal scale both within and between pairs, the Wilcoxon test should be used. If the interval measurement is achieved, the Randomization test should be used provided N is sufficiently small so as to make it computationally feasible.

In case of k related samples, if the observations are recorded in nominal scale, Cochran Q test should be used. When the data are recorded in ordinal scale or in form of scores which can be orderded, then Friedman test may be used when the number of treatments is 5 or more and Likelihood Ratio test may be followed when the number of treatments is 4 or less because in this method the computation involved is quite heavy for large number of treatments.

In case of independent samples, Fisher-Exact Probability test, χ^2—test, Median test, Mann-Whitney test, Kolomogorov-Smirnov test and Kruskal-Wallis test, may be used. In case of two independent samples, Mann-Whitney test is quite useful where as for $k>2$ samples, Kruskal-Wallis test is recommended.

Sensory evaluation provides information on various qualities of the product which may be utilised for quality control, quality improvement or development of new products etc. In sensory evaluation, observations are generally in ordinal scale or nominal scale and mostly non-parametric techniques are applicable in analysing the data.

Bradley-Terry model for paired comparisons may be used for sensory evaluation. The model provides.

$$P(T_i>T_j)=\frac{\pi_i}{\pi_i+\pi_j}$$

Likelihood estimation procedure may be used for likelihood ratio test. Rao and Kupper (1967) generalised this model to permit ties. Sadasivan and Rai (1973) modified this model for fractional paired

comparisons. Pendergrass and Bradley (1960) introduced a model for triad comparisons. The probability of selecting T_i over T_j and T_k is given by

$$P(T_i > T_j > T_k) = \frac{\pi_i^2 \pi_j}{\triangle_{ijk}}$$

where $\triangle_{ijk} = \pi_i^2(\pi_j + \pi_k) + \pi_j^2(\pi_i + \pi_k) + \pi_k^2(\pi_i + \pi_j)$.

Rai (1971) modified this model and used it for fractional triad comparisons. Durbin's model (1951) for B.I.B.D. may also be used for sensory evaluation.

A non-parametric method of analysing groups of experiments in r.b.d. has been given by Rai and Rao (1980) where only the ranks of the observations are used. The procedure is quite simple and it can be used to wide range of data. This method is valid even for the situation where error variances are heterogeneous and interaction absent for which no parametric test exists.

4. Non-Parametric Inference in Sample Surveys for Categorical Data

BY

RANDHIR SINGH

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In case of categorical data, quite often we wish to estimate the total number, the proportion or percentage of units in the population which possess some characteristic or attribute or which fall into some defined class. We may further require comparisons between such proportions or we may require to examine whether these proportions have undergone some change over time. Some times it may be of interest to study some characteristics of these units belonging to different classes and examine whether such characteristics differ from class to class or there has been any change in the characteristics of these classes over time.

In such situations often it may happen that the number of units falling into different classes may be small and as such the assumption of normality may not be valid for further analysis of such proportions

or domain means. In the present investigation, we present non-parametric test statistics for some situations met in practice insample surveys namely

(*i*) To compare the proportions or means in different classes, and

(*ii*) To examine whether there has been some change in proportions or means over time.

5. Non-parametric rank test for statistical analysis of field experiments.

BY

T.J. KHATRI AND R.M. PATEL

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The field experiments, with two way classifications in Agricultural research are analysed by the parametric technique, known as analysis of variance. The usefulness of the parametric test depends upon the validity of the assumption rhat (1) the treatments and environmental effects are additive and (2) the experimental errors are normally and independently distributed with mean, *O* and variance σ^2. Failure of any of these assumptions will impare to some extent the utility of the test. The results of insecticidal and pesticidal experiments in terms of intensity of damage due to pests and diseases, which are usually in percentages, do not satisfy these assumptions.

The data in respect of 125 field experiments with more than two treatments, conducted in r.b.d. were analysed both by Analysis of variance technique and non-parametric Friedman method. It was observed that the probability level of more than 73 per cent of the experiments yielded by the two methods are essentially the same. Thus for randomised block design with more than two treatments, the analysis by rank method given by Friedman may be utilised.

6. Application of Non-Parametric Inference Techniques in studying the Yield-Survival Relationship in Dairy Cattle

BY

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The present article deals with a study of the statistical aspects of survival of cows and buffaloes in the herd and their relationship

with their first lactation performances. Since in the present case the survival times are only recorded in term of the number of successfully completed lactations and more so with any knowledge of the survival distribution so the usual parametric techniques are not directly applicable. Thus, in this paper, the use of non-parametric inference techniques applicable to discrete observations is highlighted in particular to examine culling pattern of different crossbreds over the lactation, compare their survival distributions, study the relative culling rates, examine the yield-survival relationship at different stages of life of an animal.

In order to compare the culling patterns of different crossbreds having different level of foreign blood, the probability of survival to different orders of lactation have been studied in detail with the help of the two survival functions viz. survivorship function and hazard function. The estimates of the median survival times have also been worked out. The culling patterns of the two categories of animals are also tested statistically for the significant differences with the help of the Non-parametric tests viz. Mantel and Haenszel's chi-square test. The estimates of the relative comparison for the culling pressure or relative culling rates are obtained by considering the proportional hazard model. The relative culling rates are also tested for the significance departure from unity. A logistic model due to cox is fitted to study the relationship between probability of survival and milk yield alongwith other explanatory variables. Since this model is not linear in parameters so the iterative technique using Newton-Raphson method is employed. The use of second approach i.e. Cox's Regression model for the exact survival times instead of the response variable as dichtomous form, for studying the relationship between survival times and explanatory variables is also highlighted.